Chapter 3 – Modelling Lubricated Bearings in a Flexible Multi-Body Dynamic Environment

# Preface

Modelling electrified powertrains in flexible multi-body dynamic (FMBD) environments can enable substantial cost and time savings for automotive manufacturers due to a reduced need for physical prototyping. With increasing complexity and operational speeds of these systems, the accuracy at component level is paramount. Bearings are crucial structural components, and their dynamic response significantly affects the behaviour of the interconnected structures.

Modern electrified motors and transmissions operate at considerably higher speeds and lower loads than conventional powertrains. This leads to much higher lubricant entrainment velocities at the roller-race conjunction of the bearings. Consequently, the film thickness can be of the same order of magnitude and often exceed that of the contact deformation predicted by dry Hertzian assumptions; hence, dry analyses are no longer valid.

This chapter presents a flexible dynamic model with lubricated bearings operating under conditions representative of electrified vehicle powertrains. The multi-physics approach accounts for the tribological phenomena at the roller-race conjunction and models their effect on shaft-bearing system dynamics. This is achieved through explicit coupling of a non-linear lubricated bearing model within a flexible system level model. A flexible shaft is supported by two cylindrical roller bearings in a commercially available flexible multi-body software. The kinematic behaviour of the bearing races at each step of the dynamic analysis is passed to a separate lubricated component level bearing model during the simulation. A contact slicing method is employed to calculate the reaction forces of the individual rolling elements based on the contact deflection, with implicit inclusion of the elastohydrodynamic (EHL) lubricant film. Resultant forces on the races are returned to the system level model and the equations of motion are solved. Excitation forces representative of gear pair forces and electric motor torque fluctuations in an electrified vehicle transmission are applied to the shaft

Results in time and frequency domains have been analysed, as well as tribological quantities such as elastohydrodynamic (EHL) pressure, film distribution and lubrication regime. The EHL film is shown to enhance contact deflection, increasing contact forces and total bearing stiffness as rotational speeds increase. For lightly loaded cases at 25 000 rpm, the film reaches a thickness of 4.62 µm. Entrainment of the lubricant increases the contact deflection, causing the contact stiffness to increase non-linearly with speed. Moreover, the contact stiffness is up to 27.3 % times greater in the lubricated model compared to the dry assumption; shifting the resonant frequency of the system higher in the speed range. This highlights the necessity of this multi-physics tribodynamic approach for future high-speed powertrain modelling.

Nomenclature:

|  |  |
| --- | --- |
|  | Asperity apparent contact area (m2) |
|  | Apparent contact area (m2) |
|  | Acceleration (m.s-2) |
|  | Half-length of the contact (mm) |
| C | Radial clearance (µm) |
|  | Solid specific heat capacity (J.kg -1.K-1) |
|  | Diameter of roller (mm) |
|  | Pitch diameter (mm) |
|  | Equivalent (reduced) elastic modulus (Pa) |
|  | Radial load in x-direction (N) |
|  | Radial load in y-direction (N) |
|  | Total friction (N) |
|  | Boundary friction (N) |
|  | Viscous friction (N) |
|  | Ball Pass Frequency of Outer Race (Hz) |
|  | Ball Pass Frequency of Inner Race (Hz) |
|  | Shaft Rotational Frequency (Hz) |
|  | Dimensionless equivalent geometry (-) |
|  | Central film thickness (m) |
|  | Stiffness (Nm-1) |
|  | Lubricant thermal conductivity (W.m-1.K-1) |
|  | Roller length (mm) |
|  | Exponent of localized deflection (-) |
| N | Number of rolling elements (-) |
|  | Contact pressure (Pa) |
|  | Average pressure at the apparent contact (Pa) |
|  | Radius of inner race (mm) |
|  | Equivalent radius of contact (mm) |
|  | Speed of entraining motion (m.s-1) |
|  | Dimensionless speed parameter (-) |
|  | Velocity (m.s-1) |
|  | Contact load (N) |
|  | Dimensionless load parameter (-) |
|  | Asperity load (N) |
|  | Displacement in x-direction (m) |
|  | Conjunction x-coordinate (-) |
|  | Displacement in y-direction (m) |
|  | Conjunction y-coordinate (-) |

Greek Symbols:

|  |  |
| --- | --- |
|  | Angular position (rad) |
|  | Pressure viscosity coefficient (m2.N-1) |
|  | Contact Deflection (m) |
|  | Average asperity tip radius of curvature (m) |
|  | Stribeck parameter (-) |
|  | Atmospheric lubricant dynamic viscosity (Pa.s) |
|  | Lubricant dynamic viscosity (Pa.s) |
|  | Lubricant density (kg.m-3) |
|  | Atmospheric lubricant density (kg.m-3) |
|  | Solid density (kg.m-3) |
|  | Composite surface roughness (m) |
|  | Boundary shear strength of asperities (-) |
|  | Eyring stress (Pa) |
|  | Angular velocity of cage (rad.s-1) |
|  | Angular velocity of inner race (rad.s-1) |
|  | Angular velocity of shaft (rad.s-1) |
|  | Relaxation factor (-) |
|  | Asperity density (m-2) |

# List of Figures

[Figure 1 - Flowchart of models 7](#_Toc103805642)

[Figure 2 - Connection pins and degrees of freedom 11](#_Toc103805643)

[Figure 3 - Output port Demux blocks and degrees of freedom 12](#_Toc103805644)

[Figure 4 - Input port Mux blocks and degrees of freedom 12](#_Toc103805645)

[Figure 5 - Simulink model layout 13](#_Toc103805646)

[Figure 6 - Bearing schematic 14](#_Toc103805647)

[Figure 7 - Lubricated Roller-Race Contact 14](#_Toc103805648)

[Figure 8 - Validation of slicing technique used in the model against experimental data by de Mul et. al [20] and the Sophisticated non-Hertzian Technique [26] 16](#_Toc103805649)

[Figure 9 – Electric Hub Motor Excitation Model 18](#_Toc103805650)

[Figure 10 - PMSM Torque Profile 19](#_Toc103805651)

[Figure 11 - Rolling Element Contact Stiffness - Dry vs Lubricated Operating Envelope 21](#_Toc103805652)

[Figure 12 - Rolling Element Contact Force - Dry vs Lubricated Percentage Increase 22](#_Toc103805653)

[Figure 13 - Inner Race Stiffness - Dry vs Lubricated Operating Envelope 22](#_Toc103805654)

[Figure 14 - Inner Race Displacement - Dry vs Lubricated Operating Envelope 23](#_Toc103805655)

[Figure 15 - Inner Race Dry vs Lubricated Acceleration Operating Envelope 24](#_Toc103805656)

[Figure 16 - Film Thickness vs Contact Force 12 000 rpm 25](#_Toc103805657)

[Figure 17 - Film Thickness vs Contact Force 25 000 rpm 25](#_Toc103805658)

[Figure 18 - Greenwood Regimes for Contact Conditions at 4 000, 8 000 and 12 000 rpm 26](#_Toc103805659)

# List of Tables

[Table 1 - Bodies in System Level Model 8](#_Toc103805660)

[Table 2 - Joints in System Level Model 8](#_Toc103805661)

[Table 1 - Pinion Geometry 19](#_Toc103805662)

[Table 2 - Gear Geometry 19](#_Toc103805663)

[Table 3 - Bearing Specification 20](#_Toc103805664)

[Table 4 - Lubricant and Material Properties 20](#_Toc103805665)

# Introduction

Operating conditions of roller bearings in modern EV powertrains require dynamic modelling to capture system transience such as time-varying input forces, acceleration, and eccentricity. Early quasi-static bearing models [1], [2], [3], [4], are only applicable under steady state operating conditions, however the static equilibrium solutions [5] ,[6] ,[7] , are of use to calculate load-deflection and individual element loading within dynamic models. Simplified 2 degree of freedom dynamic models [8] consider purely in-plane motion of rolling elements in the radial and lateral directions of the bearings for investigation of frequency response to defects [9], varying compliance effect [10], and radial loading affects [11]. These models increase in complexity up to 5-DOF to observe moment loading and centrifugal effects [12] [13]. These analyses assume a dry contact between rolling elements and races, which was assumed valid under the elastohydrodynamic regime of lubrication. This, however, neglects the effect of the lubricant film thickness in the contact mechanics and thus underestimates the contact deflection and hence load. Based on the experimental and numerical findings [14], it is clear that the lubricant film in the roller bearings operating at high speeds must be implicitly included in dynamic analyses.

Previous dynamic models accounting for lubricant films have been decoupled into two stages. The first is a classic dry Hertzian contact analysis of the roller-raceway contact under cyclic variation of the geometric bearing centre [15]. The contact load yielded from dry analysis is then used to calculate central film thickness. This does not implicitly account for the lubricant film on prevailing contact deflection and load. Rahnejat and Gohar [16] later coupled the film with the contact analysis in a component level dynamic analysis, resulting in more reasonable bearing vibration amplitudes than the de-coupled analyses. Mohammadpour et al. [17] employed implicit tribodynamic analysis and utilised a full numerical elastohydrodynamic analysis explicitly. In this analysis, input shaft speeds of 209 rad/s resulted in much slower entrainment velocities than in EV case studies. Existing lubricated models are low speed [18], or only model the bearings at component level. These studies do not quantify the effect that the lubricant film has on system level stiffness in a flexible model.

In this study, an explicitly coupled simulation approach is used to combine a lubricated bearing model within a system level FMBD model. The kinematic behaviour of the shaft at each time step of the dynamic simulation is passed to the bearing model. A contact slicing method [5] is employed to calculate the reaction forces of the individual rolling elements based on the roller-race contact deflection [19]. The total deflection is influenced by the thickness of the EHL film within the contact, which is implicitly included within the analysis through an iterative procedure. Resultant race forces are returned to the system level model and the equations of motion are solved at each time step. Comparisons are made between modelling the bearings as dry and lubricated. Dynamic results including frequency spectra, force magnitudes, and stiffness variations have been obtained for dynamic loading conditions and shaft speeds up to 25 000 rpm.

# Methodology

An explicit co-simulation methodology combines a system level model of a flexible shaft and rigid housing with component level models of the lubricated bearings. Operating conditions such as rotational speed and external forces are defined in the system level mode. Time step, iteration accuracy and simulation length are also defined in this model. Material, geometric, and rheological properties of the bearings are defined in the component level model. Kinematic conditions from the system level model are passed to the component level model at each time step. For each individual rolling element, the non-linear force-deflection relationship is employed in conjunction with elastohydrodynamic film calculations to compute the contact reaction force between the roller and race. The resultant forces on the inner bearing race are then returned to the system level model where the equations of motion are solved, and the time step is advanced. The workflow of these models is shown in **Error! Reference source not found.**.

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Figure 1 - Flowchart of models

## System Level Flexible Model:

### Model Layout

The system level model consists of a flexible 50 mm diameter shaft, supported by two cylindrical roller bearings in a rigid housing. The cylindrical roller bearings act as interference elements between the shaft and housing. The shaft is constrained to lateral degrees of freedom in this study. This permits lateral motion in both vertical, , and horizontal, , directions, and rotation about the -axis. External load is applied at the shaft centre. This can be applied as a time-varying input force, to simulate gear mesh excitation, or as a static load. Rotational speed is input as a boundary condition. Information on the bodies and joints within the model are outlined in Table 1 and Table 2.

In typical operation containing flexible structures, it is possible for both inner and outer races of a rolling element bearing to move when subject to load. For this analysis, however, it is sufficient to fix the outer race in space and consider only the displacement of the inner bearing race [20]. The housing in this study is treated as a rigid body of infinite stiffness, therefore the race dynamics of the bearing are influenced only by the motion of the flexible shaft in the model. The loading on the inner race is reacted by the rolling elements on the inner raceway. This must therefore be solved to achieve an equilibrium.

Table 1 - Bodies in System Level Model

|  |  |  |
| --- | --- | --- |
| **Body** | **Type** | **Active DOF** |
| Shaft | Flexible | T2, T3, R1 |
| Housing | Rigid | None |
| Motor | Rigid | None |

Table 2 - Joints in System Level Model

|  |  |  |
| --- | --- | --- |
| **Joint** | **Type** | **Connected Bodies** |
| Bearing 1 | Link to MATLAB® (Lubricated Cylindrical Roller Bearing) | Shaft & Housing |
| Bearing 2 | Link to MATLAB® (Lubricated Cylindrical Roller Bearing) | Shaft & Housing |
| Coupling | ROTX – Rotational Coupling | Shaft and Motor |

### Governing Equations

Within the model, the shaft is treated as a body having linear elastic behaviour and the housing is treated as rigid. The bearings are modelled via non-linear contact forces acting between the shaft and housing.

The shaft is represented by a condensed finite element model and is discretized into a sufficiently high number of partial masses. The total elastic deformation of the shaft is represented by translational displacements and rotational distortion components of these individual partial masses. The mathematical modelling used in the FMBD solver is based on Newton’s equations of motion and Euler’s equation of angular momentum.

|  |  |
| --- | --- |
|  | [1] |
|  | [2] |

where and represent the mass and inertia tensors of the partial masses, . The vectors of displacement and angular velocity are represented by and respectively and are related to the centre of gravity of each partial mass. The force and moment vectors, and , must be fulfilled for all partial masses in the shaft.

The matrices of the shaft are solved in a body-fixed coordinate system and transformed into the relative (reference) coordinate system using vector rotations of the origin of the reference (body fixed coordinate system) relative to the origin of the absolute coordinate system.

The combination of displacement and rotations of the body takes the form:

|  |  |
| --- | --- |
|  | [3] |

where represents the block-diagonal mass matrix of the shaft, consisting of the sub-matrices that make up each partial mass of the full shaft. is itself a block diagonal matrix, containing the mass of the sub-body, , which is multiplied by a 3x3 unit vector, , and the tensor of inertia that corresponds to that partial body, .

|  |  |
| --- | --- |
|  | [4] |

in equation 3 represents the second derivative of the displacement vector of all partial masses, . Each element of this vector has, itself, 6 elements associated with it that represent the 6 degrees of freedom – 3 rotational and 3 translational ().

The sub-vectors of force, , contain the forces and moments acting on each partial mass. These are split into a sum of internal force terms, , external force terms, , and non-linear inertia terms, . As with the partial mass terms, these also have 6 elements each, representing the 6 degrees of freedom:

|  |  |
| --- | --- |
|  | [5] |

where each component of force, is evaluated using the linear-elastic approach.

|  |  |
| --- | --- |
|  | [6] |
|  | [7] |

where and k represent the damping and stiffness coefficients, respectively.

Grouping the damping and stiffness coefficients into one matrix gives the equation of motion after rearrangement. This equation represents the behaviour of the total system of rigid partial masses that make up the shaft, and considers both general global motion and small body motion (vibrations):

|  |  |
| --- | --- |
|  | [8] |

The vector of external forces and moments, , is the sum of excitation forces, , and external loads, .

|  |  |
| --- | --- |
|  | [9] |

External loads and moments applied to the shaft are determined functions given in time and can be input as time-varying or static loads on the system. The non-linear excitation term, , is provided by the component level bearing model.

## Co-Simulation Methodology:

Nodal displacements, , and velocities,, at the bearing locations are output from the dynamic model and used as boundary conditions within a lubricated component level bearing model at each time step of the simulation. This model returns resultant forces and torques on the inner race of each bearing, which are then used to solve the equation of motion (equation 8) within the dynamic model.

### Co-Simulation of Coupled Models

A coupled simulation approach is employed to implement the lubricated components level bearing model within the flexible multi-body dynamic (FMBD) environment. AVL EXCITETM contains integrated “Link to MATLAB” functionality, whereby joints within the model can be replaced with a user function created in MATLAB Simulink®.

### Link to MATLAB

#### Connected Degrees of Freedom

Connections are created between bodies in the FMBD software using pins. Pins are assigned to specific nodes on the shaft and housing. The pins transmit a 6-element vector that describe the 6 DOFs (degrees of freedom) of the connection point. The first 3 elements translational degrees of freedom, whilst the latter represent rotational degrees of freedom. This are concatenated into a single vector that is then passed to MATLAB®.



Figure 2 - Connection pins and degrees of freedom

Two vectors are transferred from EXCITETM Power Unit to MATLAB®: one containing displacement information in all 6 DOFs and the other containing velocity information. Force vectors of equal size are then computed within the component level model in MATLAB® and returned to EXCITETM Power Unit.

The connection to MATLAB® is facilitated via an S-function, therefore a Simulink® model is required. A generic block developed by AVL, the EXCITETM Power Unit Simulink block, is used for this purpose:

#### Output Port

Output data from the FMBD model is organised in vector format, of size 6n where n is the total number of connections being passed from MATLAB® to EXCITETM Power Unit. To access data for each connection, and then split this into specific degrees of freedom, two levels of Demux elements are needed.

1. The first level of Demux splits the output vector into n number of pins, of size 6 to represent the degrees of freedom at each pin.
2. The second level of Demux blocks splits the 6-element vector for each pin into the 6 scalar values that represent the data for each degree of freedom. These can then be used as inputs to the MATLAB® function block within the Simulink® model.

Diagram

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Figure 3 - Output port Demux blocks and degrees of freedom

#### Input Port

At the input port to the EXCITETM Power Unit Simulink block, a vector of 6n elements must be passed. This requires two Mux blocks: the first to combine the degrees of freedom of each pin, and the second to combine all of these connection vector into a single vector that is passed to EXCITETM Power Unit.

Diagram

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Figure 4 - Input port Mux blocks and degrees of freedom

#### Simulink Model

To integrate the lubricated bearing model into the Simulink® model, the MATLAB® Function block is used. The inputs to the block are defined at the beginning of the function, and the appropriate degrees of freedom are connected to the block in the order of which they appear in the script. The function blocks and hence lubricated bearing models are highlighted in Figure 5, as well as the ports and the demux blocks that split the degrees of freedom.

Diagram, schematic

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Figure 5 - Simulink model layout

The coupled simulation supported by EXCITETM Power Unit is explicit in nature. This means that there is no iteration at each time step between the dynamic solver and the bearing model. A sufficiently fine time step is therefore required to prevent numerical divergence of results due to error accumulation. It was found that a time step of 1e-6 s was fine enough to ensure numerical convergence, whilst also remaining computationally efficient.

## Lubricated Component Level Model:

### Contact Force-Deflection Relationship:

The displacement and velocity vectors from each node connecting the shaft to the bearings, and respectively, are split into 6 degrees of freedom. For the lateral DOF model, translations in and are considered, as well as angular displacement around the rotational axis, . A schematic of the bearing is shown in Figure 6.

A picture containing telephone, electronics

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Figure 6 - Bearing schematic

Between the roller and raceways, under sufficient load, the pressures in the non-conformal contact are high enough to cause elastic deformation of the surfaces and a significant increase in lubricant viscosity. This leads to the generation of an EHL contact. The stiffness of the EHL film is typically 1-2 orders greater than the stiffness of the contacting bodies. As a result of this, the stiffness of the film can be neglected [21] [22], and it can be modelled as a rigid element that is present between roller and race.

The contact deformation, , is therefore a function of the displacement of the inner bearing race, angular position of the roller, , thickness of the EHL film, , and any clearance or radial preload, , within the bearing [16] [17]:

|  |  |
| --- | --- |
|  | [10] |

Figure 7 demonstrates this more clearly. The total contact deformation is the summation of the central film thickness and the material deformation, , predicted from the dry-hertzian contact assumption.

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Figure 7 - Lubricated Roller-Race Contact

In the case of a rolling element, a cylindrical body of finite length, the contact problem is non-Hertzian. The surfaces cannot be modelled as locally quadratic due to the presence of crowned (rounded) edges [23]. For the point contact case, Hertz [24] proposed an analytical solution for the load-deflection relationship – however, no such relationship was provided for the line contact. The most widely used of these techniques is the contact slicing technique. Whilst this does not reflect edge stress concentrations, these stresses are only distributed over a small area and hence can be neglected for the purpose of force equilibrium [25]. In general, this technique is favoured for its simplicity, speed, and sufficient accuracy.

Andreason [5] developed the slicing technique for modelling non-Hertzian line contacts. The roller is sliced into smaller sections along its length. The contact load intensity at each slice is obtained as the one on the roller should the roller be subject to a contact compression equal to the one occurring on the slice considered over its entire length.

Modelling the roller-race contacts as a line contact between a cylindrical roller and a flat surface, Lundberg’s [19] expression between contact force per unit length, , and deformation, , was used. This assumes a uniform pressure distribution along the length of the contact, and an elliptical one across it. This neglects side leakage along the contact () due to the contact dimensions in this direction being much larger than dimensions across it ( ). This is valid apart from the small regions at the edges of the contact.

|  |  |
| --- | --- |
|  | [11] |

where is the equivalent elastic modulus of the two materials and is the active length of each slice along the roller.

From equation 11, an equation based on empirical data can be approximated to calculate contact forces per unit length of an individual slice along the roller-race contact. This is valid if there is no separation of the bodies, i.e.. the contact deformation does not become negative.

|  |  |
| --- | --- |
|  | [12] |

where represents the slice number. It is assumed that total contact deflection is shared equally between inner and outer races.

The application of this slicing technique within the roller bearing model was validated against open literature. de Mul et al. [20] compared results obtained from an experimental rig with numerical results calculated using bother the approximate slicing technique and the sophisticated non-Hertzian technique [26]. By replicating the geometry of the test bearing used in their analysis, the application of Andreason’s slicing technique within the model used for this analysis was validated with good agreement. Results of this validation within a realistic loading region are shown in Figure 8.

Chart, scatter chart

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Figure 8 - Validation of slicing technique used in the model against experimental data by de Mul et. al [20] and the Sophisticated non-Hertzian Technique [26]

The damping force for each roller is obtained as factor of contact stiffness and contact penetration velocity [27]. This is defined as:

|  |  |
| --- | --- |
|  | [18] |

where K is the contact stiffness, and the damping factor, , is in the range of as reported by Krämer [27].

The total contact load and moment are obtained by summing the contributions from all loaded slices:

|  |  |
| --- | --- |
|  | [13] |
|  | [14] |

with being the slice length. This simple method is a much faster way of calculating contact load and moment than more sophisticated methods by de Mull [26].

At each time step of the analysis, these calculations are performed for each individual roller in the complement. The total bearing force acting on the inner race is solved by splitting the total contact force on each roller into its components and summing their contributions.

|  |  |
| --- | --- |
|  | [15] |
|  | [16] |

### Implicit Tribological Model

For a sufficiently loaded bearing, contact pressures are such that the lubrication regime is elastohydrodynamic. In a suitably preloaded bearing, this film contributes to the total deformation at the contact and hence contact force. Misalignment along the length of the rollers is not considered due to the high stiffness of the shaft and bracket, hence a 1-dimensional analysis for EHL is sufficient [13].

The extrapolated central film thickness for a line contact is therefore obtained [28] from:

|  |  |
| --- | --- |
|  | [21] |

where the following dimensionless parameters are used:

|  |  |
| --- | --- |
|  | [22] |

Assuming pure rolling due the sufficient preload, the speed of entraining motion is given by [29] [30]:

|  |  |
| --- | --- |
|  | [23] |

Due to the dependency of load on film thickness, an iterative approach is performed to calculate the contact force. Convergence criteria for the EHL film must be met at each time step of the simulation before the bearing forces are returned to the system level model and equations of motion are solved:

|  |  |
| --- | --- |
|  | [24] |

where represents the iteration number.

Contact conditions between the inner and outer races are assumed the same, despite slight changes in their contact geometry. Entrainment velocity is equal at each contact, which is the governing parameter for the lubricated contact force differences. The stiffness and damping of the EHL film is neglected due to its rigid-like stiffness which is several orders of magnitude higher that the Hertzian contact [21] [22].

## Representative Excitation Methodology

The system level model is decomposed, with excitation forces calculated externally before being applied within the model. A separate electrified transmission model is used to generate realistic excitation forces and torques from a spur gear pair and a permanent magnetic synchronous motor (PMSM). This system represents the first stage of an electric hub motor used in automotive applications.

Diagram

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Figure 9 – Electric Hub Motor Excitation Model

Radial and tangential gear pair forces at the pinion centre, as well as torque fluctuations of the electric motor are extracted to be used as inputs to the lubricated bearing model. All bodies in this system were modelled as rigid, so that structural excitation forces did not contribute to the resultant forces at the pinion.

Peak torque of the motor is 68 Nm, and the system operates at peak power through all speed increments up to 25 000 rpm. The torque transfer through the gear pair reduces as speed increases due to the torque profile of the PMSM, as shown in Figure 10. Stator tooth forces from the PMSM are neglected in the model due to their minimal contribution to lateral forces once resolved. For input to the model, radial and tangential forces are simplified to sinusoidal inputs of the same magnitude and frequency of the gear pair at different speeds. Torque ripple from the motor is simplified using the same method.

Chart, line chart

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Figure 10 - PMSM Torque Profile

Gear pair geometry is shown in Table 3 and Table 4.

Table 3 - Pinion Geometry

|  |  |
| --- | --- |
| Number of teeth | 17 |
| Normal Module | 0.004 m |
| Normal pressure angle | 20 ˚ |
| Helix angle at pitch circle | 0 ˚ |
| Active tip diameter | 0.076 m |
| Active root diameter | 0.065 m |
| Width | 0.035 m |

Table 4 - Gear Geometry

|  |  |
| --- | --- |
| Number of teeth | 51 |
| Normal Module | 0.004 m |
| Normal pressure angle | 20 ˚ |
| Helix angle at pitch circle | 0 ˚ |
| Active tip diameter | 0.212 m |
| Active root diameter | 0.202 m |
| Width | 0.030 m |

Light preload was applied to the bearings to maintain contact throughout the rollers orbit. In practice, preload is applied to prevent skidding, however excessive preload can lead to frictional losses and wear. Excessive clearance can lead to chaotic behaviour [31]. Bearing geometry is detailed in Table 5. Rheological and material properties are detailed in Table 6.

Table 5 - Bearing Specification

|  |  |
| --- | --- |
| Inner Race Bore | 25 mm |
| Pitch Diameter | 60 mm |
| Roller Diameter | 8.8 mm |
| Roller Length | 15 mm |
| Number of Rollers | 17 |
| Radial Interference | 2 µm |

Table 6 - Lubricant and Material Properties

|  |  |
| --- | --- |
| Pressure Viscosity Coefficient () | 2.1 10-8 Pa-1 |
| Atmospheric lubricant dynamic viscosity () | 0.08 Pa.s |
| Lubricant inlet density () | 833.8 kg.m-3 |
| Modulus of elasticity of contacting solids | 210 GPa |
| Poisson’s ratio of contacting solids | 0.3 |

# Results and Discussion

## Time-Domain Results of Operating Profile

A quasi-dynamic speed sweep has been performed from 1 000 – 25 000 rpm. Simulations are performed every 1 000 rpm, refined to 250 rpm intervals throughout a period of resonance between 12 000 rpm and 14 000 rpm. Operating envelopes have been generated by plotting the maximum and minimum values from the steady state signals at each speed interval.

Conjunction level results are taken from an individual roller and its contact with the inner raceway. Component and system level results are taken from the geometric centre of the inner bearing race, corresponding to the bearing seat on the shaft. The following figures represent results in the direction, the largest component of excitation due to the tangential force from the gear meshing event.

Chart, line chart

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Figure 11 - Rolling Element Contact Stiffness - Dry vs Lubricated Operating Envelope

Contact level results show a difference in contact stiffness between the dry and lubricated models. The dry model follows torque profile of the motor, with stiffness decreasing as the contact forces reduce. The period of resonance leads to larger amplitude excitation of the shaft, resulting in an increase in contact stiffness due to the force-deflection non-linearity. The lubricated model shows an increase in contact stiffness throughout the speed sweep. This is due to the higher levels of deformation at the contact as lubricant is entrained, increasing with entrainment velocity. The contact stiffness in the lubricated model under the same operating conditions is 27.3% greater at 25 000 rpm.

Chart, line chart

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Figure 12 - Rolling Element Contact Force - Dry vs Lubricated Percentage Increase

Peak contact force has been compared between both models. The percentage increase between the dry and lubricated models is shown in Figure 12. This more clearly shows the disparity between both models at the contact, with the largest difference being 10 times at maximum speed. During resonance, the inner race force reaches a peak of 1514 N, resulting in surface deformation magnitudes of 0.92 µm and 3.81 µm at the dry and lubricated conjunctions respectively. As noted in previous works [14], higher loads lead to a greater surface deformation to film thickness ratio, causing the percentage difference between dry and lubricated models to reduce. Once the loads reduce as speed increases, the percentage increase continues to rise.

Chart

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Figure 13 - Inner Race Stiffness - Dry vs Lubricated Operating Envelope

The total bearing stiffness (Figure 13) is a function of all contact stiffnesses between the elements and raceways. These vary non-linearly with force, resulting in the total bearing stiffness varying with load. For the dry model, this is clearly demonstrated, with greater total bearing stiffness at the peak of the resonance due to greater bearing forces. This does not, however, capture the change of bearing stiffness with speed; the average bearing stiffness does not change. The lubricated bearing is not only stiffer than the dry bearing, but this stiffness also increases with speed. This is shown by the gradient of the operating envelope.

Chart

Description automatically generated

Figure 14 - Inner Race Displacement - Dry vs Lubricated Operating Envelope

Due to the greater total stiffness of the bearing, the shaft displacement of the lubricated model is lower both on average and peak to peak for the same applied force in comparison to the dry model. Through the period of resonance, the large inner race forces result in roller-race separation of unloaded rollers within the dry model. This leads to greater shaft displacement as the inner race moves into this region of zero reaction force. Larger magnitude of inner race displacement due to separation of rolling elements from their contact. as deformation rises due to unloaded regions of the dry bearing.

Chart, diagram

Description automatically generated

Figure 15 - Inner Race Dry vs Lubricated Acceleration Operating Envelope

The acceleration peak of the resonance occurs at 12 500 rpm (3542 Hz) for the lubricated model as opposed to 12 250 rpm (3470 Hz) for the dry model. This shift in natural frequency of the system indicates a stiffer overall system. The magnitude difference between dry and lubricated models can also be attributed to unloaded regions of the dry bearing. The contact deformation arising from the loading of the inner race is sufficient to cause rollers geometrically opposite those experience loading to become separated from their own contacts. Contact is lost between the roller and raceway, leading to zero contact stiffness. The inner race moves into this region until it is reacted by a contact force once again.

Results for an individual roller in its orbit around the bearing are shown in Figure 17 and Figure 16. On both figures, the gear mesh frequency is observed superimposed on the ball pass frequency as the roller enters and exits the loaded region. Due to sufficient preload within the bearing, the roller maintains contact with the raceways throughout its orbit, shown by non-zero contact force values. Peak force values occur as the roller passes through the centre of the loaded region; coinciding with the lowest film thickness values as the EHL film is compressed.

The effect of the rigid EHL film on the contact force is clear when comparing both speed cases at 12 000 rpm and 25 000 rpm (Figure 17 and Figure 16 respectively). Even though the excitation force at 25 000 rpm is much lower due to the torque profile of the motor and absence of resonance, the contact forces are still greater than at 12 000 rpm. This is due to the EHL film acting as an interference element between the roller and raceway. The influence of the resonance at 12 000 rpm can be clearly seen when observing the peak-to-peak fluctuations of the force and film values. The much higher amplitude oscillations of the shaft cause greater variations in the material deformation and hence contact force.

Diagram

Description automatically generated

Figure 16 - Film Thickness vs Contact Force 12 000 rpm

Chart

Description automatically generated with medium confidence

Figure 17 - Film Thickness vs Contact Force 25 000 rpm

## Greenwood Regimes of Lubrication

For the case of the lubricated bearing with no clearance, contact deformation occurs between each roller and race around its entire orbit due to the presence of the lubricant film. Under these conditions, the lubricant regime is elastohydrodynamic for the full cycle. If the bearing has clearance and force magnitudes are large enough to cause separation of the raceways, then the regime of lubrication will transition from EHL to hydrodynamic.

A method of modelling the regime of lubrication that the contact is operating under is using Greenwood regimes of elastohydrodynamic lubrication [32]. The charts display the physical effects instrumental to EHL formation under isothermal conditions: viscosity rise due to pressure and elastic deformation of the surface.

The piezoviscous elastic (PE) regime signifies the EHL regime of lubrication where contact pressures are such that elastic deformation of the surfaces and viscosity rise due to pressure increase is significant. The iso-viscous rigid (IR) regime occurs when the magnitude of elastic deformation is insignificant, and the contact pressures are low enough that viscosity rise is negligible, i.e. hydrodynamic lubrication.

Contact force per unit length and entrainment velocity for roller loading cycles at 4 000 rpm, 8 000 rpm ad 12 000 rpm were used to calculate the viscosity parameter, A, and elasticity parameter, B. The results are then overlayed on the line contact boundaries to assess how far into each region the contact operates. It is shown that as speed increases, due the greater value of contact deflection and hence force, the greater pressures result in the contact moving further into the piezoviscous elastic region of the plot.

Diagram

Description automatically generated

Figure 18 - Greenwood Regimes for Contact Conditions at 4 000, 8 000 and 12 000 rpm

# Conclusions

A methodology has been developed to implement a lubricated bearing model within a flexible system level model. The model implicitly includes the lubricant film at the roller-race contact within the bearing; something that has not, to the author’s knowledge, been reported on in open literature. Simulations have been performed up to 25 000 rpm with realistic excitation forces from a first stage reduction gear pair and electric motor. Conjunction level and system level results have been analysed to compare

Results show that the film thickness reaches 4.62 µm at 25 000 rpm. This leads to a 10 times greater contact load and hence 27.3 % greater contact stiffness between the dry and lubricated models due to lubricant entrainment and non-linear hertzian force-deflection relationship. The contribution of all rolling elements leads to a 17.24 % greater total bearing stiffness at 25 000 rpm between both models. Moreover, this stiffness is shown to change with speed due to the increasing film thickness as entrainment velocity increases; something that dry models do not account for.

This increase in total bearing stiffness leads to a change in the stiffness of the total system. By modelling the shaft as a flexible body, the influence on the change in system natural frequency is seen. The resonant peak at 12 500 rpm shifts 250 rpm higher in the lubricated model which coincides with higher frequency excitation from the gear meshing event. Understanding the effect of roller bearings on transmission stiffness is of paramount importance in automotive applications, and this change show the influence of the lubricant film on these already complex phenomena.

This work proves the necessity of multi-physics modelling of the tribology and dynamics of high-speed rolling element bearings in future powertrain modelling to ensure accurate component and system level behaviour.

# Additional Work for Chapter Completion

## Different Lubricants

* Investigate effects of different lubricants (change dynamic viscosity, density, and pressure-viscosity coefficients in the bearing model and run selected conditions).

## Artificial Neural Network Film Thickness Calculations

* Run implicit ANN models with contact data from the bearings in the system level model. Will show better accuracy over extrapolated equations at higher entrainment speeds, with lower computational demand. These models have already been developed in earlier stages of the PhD.

## Refinement of written work

* Chapter requires general refinement to the written work and layout.

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